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A TIME SERIES ANALYSIS OF OUTPUT-ORIENTED PRODUCTION EFFICIENCY IN NIGERIA AGRICULTURE

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Abstract

This paper investigates possibilities of reducing production inefficiency using same input vector. Firms mostly suffer technical inefficiency in their production. Producers are found to operate below the frontier. Hence, empirical measures of production efficiencies are necessary to determine the inefficiency level, magnitude of inefficiency reduction, and gains that could be obtained by improving performance in the sector. The study used secondary data collected from Food and Agriculture Organization (FAO) and the National Bureau of Statistics (NBS) database. The dataset covers the period 1960-2021. Variables extracted included agricultural GDP (kg), fertiliser usage (kg), agricultural labour(man-days), and number of tractors (No). The data were subjected to a unit root test for stationarity, and a stochastic frontier production/ Cost function model were applied to determine technical inefficiency of variables in the model. The result of unit root shows that series are integrated of the first order I (1). AIC criteria indicate an optimal lag length of two years, while the Unrestricted Co-integration Trace and Maximum Eigenvalue test show strong evidence of long-run relationship amongst variables. The parameters of Stochastic Frontier Production Function estimated were positively consistent with the study's a priori expectation for such variables as fertiliser (0.2634), labour *(0.3159), land (0.1846), and tractor (0. 1587). Output-oriented technical efficiency is 0.7802 (78%) and 22% is technically inefficient. A decreasing returns to scale value of (0.9226) with a scale effect of 0.9226. The sector is 58% economically efficient with cost savings of 42% and 75% allocative efficiency. This study concludes that Nigeria's agriculture suffers from production inefficiencies and this inefficiency can be reduced by using the same input levels.*

Keywords: *Cost function, efficiency, Return to scale, Scale effect, and Stochastic.*
JEL Codes: *C22, D2*

1. Introduction

Nigeria is an agrarian nation with numerous natural assets, including 960 kilometers of coastline, 68 million hectares of arable land, close to 12 million hectares of freshwater resources, and ecological diversity that enables the production of a wide range of agricultural products Arokoyo (2012). Despite these abundant resources, research reveals that agriculture's economic impact on the country has been declining (Ekpo & Umoh, 2012; Al-Hassan, 2013; CBN, 2014). Agriculture's contribution to GDP has decreased significantly over time, from 60% in the early 1960s to 48.8% in the 1970s and 22.2% in the 1980s, NBS (2014), and to only about 21.9% in 2019. World Bank, (2020). However, Nigerian agriculture has not yet developed into a well-organised and commercialised industry, Olukunle (2013). Available literature (Uma *et al.,* 2013; Ugwu & Kanu, 2012) suggests a paradigm shift away from policies that place more emphasis on agricultural output and towards approaches that are more integrated, interconnected, and target-specific. In fact, rather than increasing, the sector's productivity has decreased NBS (2014). These findings make the agriculture industry a crucial topic for research across several stages.

The sector's level of inconsistent efficiency (i.e., inefficiency) suggests that output can be increased without using more inputs. That is, by using input- or output-oriented methods, such as reducing input use, improving input quality, increasing market access, or using any other method. As a result, Technical efficiency (TE), Allocative efficiency (AE), Economic efficiency (EE), and CE empirical measurements of production efficiency are required. Several cross-sectional studies focus on particular crops and agriculture in Nigeria (Ogundari & Ojo, 2007; Oni *et al.,* 2009; Bravo-Ureta & Rieger, 1991; Battese, 1992; Djokoto, 2012; Kea *et al.*, 2016) and agriculture in general (Bravo-Ureta & Rieger, 1991; Djokoto, 2012; Kea *et al.*, 2016), and employed the stochastic frontier approach to evaluate technical efficiency. (Felix *et al.* 2021; Kumar & Paramasivam 2019) use the stochastic frontier method to evaluate technological efficiency; Ajani & Ugwu (2010), also using the frontier approach, calculated the effect of poor health on agricultural productivity. This research made the supposition that the main source of production inefficiency for businesses is technical. This suggests that producers were discovered operating below the boundary. To ascertain the levels of inefficiency, the amount of the likely drop in inefficiency, or the benefits that can be obtained by improving performance in agricultural production or the agricultural sector, empirical measurements of production efficiency are therefore required.

The study's research gap focuses on the potential for lowering production inefficiency by utilizing an output-oriented approach or strategy that uses the same input vector. The main focus of this work is a Time Series Analysis of Production Efficiency in Nigerian Agriculture: 1960–2021 (62 years) utilizing Output–Output-oriented measures.

Theoretically, this study is based on duality theory. The most essential development in the theory of production and cost is Shepherd's (1953, 1970) finding a twin relationship between production and cost function, Chambers & Quiggin (1998). Agricultural production is full of uncertainties and because of this and because economic problems connected with altering it have provided arguments for its special nature and its preferred treatment in the economy. Due to the stochastic nature of agricultural production, its production differs from other non-stochastic production. Chambers & Quiggin, (1998), opined that with a closed and non-empty input set, a production function can generate a well-behaved cost function. The latter is a twin of the former, showing a convex of inputs and free disposability of inputs. Therefore, the study is centered on the self-dual production frontier function. Adopting generalised Cobb-Douglas frontier production function (Fan 1991), allows for the use of output-oriented production efficiency, and returns to scale. Precisely, the following broad stochastic production function is considered

$$Qit = f_{jit}\ t\text{: } \alpha)\ \exp\ (V_{it} - U_{it}) \qquad (1)$$

Where

Q* is the maximum output that can be produced; K_{jit} is the ratio of observe input X_{iit} and X_{jit} at $Q^*_{it\,;}$ (*) = functional form; Q_{it} = ith farm output in time t ; X_{jit} = quantity of $_{jth}$ input ;α = vector of parameter estimated ; eir = $V_{it} – U_{it}$ composite statistical noise ; V_i defines normally and symmetric dispersed error term not quantified by farmers ;U_{ir} is the output oriented technical efficiency; U_{it} is non-negative, one sided error indicating stochastic underperformance of ith farm output as a result of technical inefficiency ;The U_{it} and Vit are expected to be independently distributed from one another

$$V_{it} = f(X_{1t}, X_{2t}, X_{3t}, X_{4t}) + V_i - U_i \quad (2)$$

The stochastic frontier production function in equation 1 is estimated as:

$$V_{it} = f(fert_{1t}, labr_{2t}, land_{3t}, trac_{4t}) + V_i - U_i \quad (3)$$

where

V_{it} is the maximum output value added to the sector, and $X_{it's}$ are the vector of inputs, $B_{i's}$ is a vector of parameters to be estimated, V_i is the two-sided normally distributed, systematic component. U_i is the one-sided efficiency component with a half-normal distribution. It is assumed to have a non-negative distribution with N~ $(0,\sigma u^2)$ is a random variable assumed to account for the existence of technical inefficiency. V_i-U_i = e_i where e_i represents the error term of the traditional deterministic production function formulation.

The estimating Stochastic frontier production function equation log-linearized is given as:

$$\text{In } Q_{it} = \text{In}\beta + b_1\text{Infert}_{1it} + b_2\text{Inlabr}_{i2t} + b_3\text{Inland}_{i3t} + b_4\text{Intrac}_{i4t} + V_i - U_i \quad (4)$$

The maximum likelihood estimates (MLE) of (4) on the assumption that V_i and U_i are independent provides estimators for the parameters bi's, the variance parameters for the one-sided U as σu^2 and for the two-sided V as σv^2. The sum of these variances gives the sigma^2 (σ^2). Hence, $\sigma^2 = \sigma_u^2 + \sigma_v^2$.

The ratio of the two standard deviations of the error terms as used by Jondrow et al., (1982) is called Lambda (λ) and $\lambda = \sigma_u/\sigma_v$. While $\gamma = \sigma_u^2/\sigma^2$. Lambda (λ) ≥1 and $0 < \gamma \leq 1$.

Applying the stochastic efficiency decomposition model to the Nigerian agriculture sector is the overall goal. Additionally, the study calculates the returns to scale in the sector, identifies the scale effect in production in the sector, estimates output-oriented production efficiency/inefficiency in the sector, examines the possibility of a cost reduction that could be achieved through improvement in production efficiency in the sector, and makes policy recommendations based on the study's findings. This project has policy importance since it will empirically measure several production efficiency/inefficiency indices and look at ways to cut costs and reduce degrees of inefficiency based on its empirical findings.

Technical efficiency, which is a key factor in determining economic efficiency (Ikram *et al.,* 2016), always entails minimal input waste, therefore, to be economically efficient, production from a given resource must be at its highest possible level (Fare & Lovell, 1978). Therefore, there is a need for modern studies and empirical data on production in Nigerian agriculture for two purposes. One is to look into the possibility of output increases with a fixed set of inputs. To get the same level of output, the vector of inputs should be significantly reduced.

Output measures the value of the total amount of goods and services in the agricultural sector. (Lipsey & Crystal, 1999). Since the output is thus measured, using a production function (Cobb. Douglas), it is possible to obtain elasticity that can be used to obtain the marginal value product (MVPs) of each of the inputs used in production. The MVP under perfect market conditions equals the price of the inputs. This can be used to gauge the resource use efficiency of the sector. With the value-added dependent variable, the emphasis is on the interpretation of the variation in value added in terms of changes in the input, Mundlak *et al.,* (2002). Mundlak *et al.,* (2002) stated that when factor prices are available the marginal rate of technical substitution between inputs can also be calculated to check the validity of the condition being upheld.

This study is based on a single equation methodology. The estimation of a single production function yields biased estimates. This is because such analysis assumes instantaneous adjustment of output to changes in the levels of inputs. This assumption results in what is known as simultaneous equation bias which is a limitation (Zellner *et al.,* 1966; Kalirajan & Flinn, 1983). According to Zellner *et al.,* (1966), this limitation can be overcome by assuming that uncertainty surrounds future output only, all prices are known at the beginning of the period of production and the technical unit of production (producer) maximizes expected profit rather than actual profit. It must be noted that in economics, the dependence of a variable on another variable is rarely instantaneous. Very often the dependent variable responds to the independent variable over or with a lapse of time (a time lag). This study is carried out with this background information in mind.

2. Methodology

2.1. Data collection

The research is supported by secondary data. The FAO and NBS databases were used to acquire the annual time series data for the Nigerian agricultural industry, and the value-added/agric. variables were used in the study. The GDP of the industry, the amount of fertilizer used per kilogram, the amount of labour used in agriculture per man-day, and the number of tractors available in the industry. The data collection spans the years 1960 through 2021.

Variables used for the model include Q = Output represented by agricultural GDP (million Naira);fert = fertiliser (million metric ton/million kg.);labr = labour (‘000 man-days); land = land (‘000 hectares); tract = capital (Tractors) (number in ’00 and ‘000) ;t. = time variable
Fertiliser: The fertiliser used represents the total quantity of nitrogen (N), potassium (P_2O_2), and phosphate (K_2O) used were stated in thousands of tons, and used to measure total commercial fertilisers input used.
Labour: Labour refers to all economically active people in agriculture every year in the nation.
Land: Agricultural land refers to the total arable land cultivated, which includes land used in temporary meadows for mowing or pasture, temporarily fallow lands, temporary crops, or land under permanent crops such as coffee, and cocoa and land under pastures, these are land used for planting herbaceous forage crops, measured in
Tractors: Measured by the total number of operational tractors in the agricultural sector

2.2. Model Specification:

Following Aigner *et al.,* (1977); and Maeusen & Van den Broeck (1977), a stochastic production function with the disturbance term composed of some parts, a systematic part (v) and a one-sided (u) component is specified for the agricultural sector of Nigeria. The analytical framework borrows from the works of Bravo-Ureta & Pinheiro (1997); and Karagiannis & Tzousvelekas (2001). The production function is specified as

$V_{it} = f(fert_t, labr_t, land_t, trac_t) + V_i\text{-}U_i$ (5)

Where: V_{it} is the value added of the sector, and $X_{it's}$ are the factors (fertiliser, labour, land, tractor) hypothesized as influencing value added in the sector.

A multiplicative production frontier for which Cobb Douglas production function as specified is assumed. The sector's technology is thus represented by a Cobb-Douglas function and the stochastic production function is given as:

$Va_{it} = \beta fert_{i1t}{}^{b1}\ lab_{i2t}{}^{b2}\ lan_{i3t}{}^{b3}\ trac_{i4t} + V_i\text{-}U_i$ (6)

Where: Va_{it} is the yearly value added of the sector. $X_{it's}$ is a vector of inputs hypothesized as the main factors in the sector. $\beta_{i's}$ is a vector of parameters to be estimated,

V_i is the two-sided normally distributed, systematic component. This captures random variations in Va_{it} due to factors outside the sector. It is assumed to be independently and identically distributed as Na $(0,\sigma v^2)$

U_i is the one-sided efficiency component with a half-normal distribution. It is assumed to have a non-negative distribution with N~ $(0,\sigma u^2)$ is a random variable assumed to account for the existence of technical inefficiency. $V_i\text{-}U_i = e_i$ where e_i represents the error term of the traditional deterministic production function formulation. The estimating SFPF equation log-linearized is given as

$\text{In } V_{it}=\text{In}\beta+b_1\text{Infert}_{iit}+b_2\text{Inlabr}_{i2t}+b_3\text{Inland}_{i3t}+b_4\text{Intrac}_{i4t}+V_i\text{-}U_i$ (7)

The maximum likelihood estimates (MLE) of (5) on the assumption that V_i and U_i are independent provides estimators for the parameters. bi's = variance parameters for the one-sided U as σu^2, and the two-sided V as σv^2. The sum of these variances gives the sigma² (σ^2). Hence,

$\sigma^2= \sigma_u{}^2+\sigma_v{}^2$. (8)

The ratio of the two standard deviations of the error terms as used by Jondrow *et al.*, (1982) is called Lambda (λ) and

$\lambda= \sigma_u/\sigma_v$. (9)

While $\gamma =\sigma_u{}^2/\sigma^2$.

Lambda (λ) ≥ 1 and $0 < \gamma \leq 1$.

Stochastic Frontier Cost Function

Following Taylor et al., (1986) the analytically derived stochastic frontier cost function (SFCF) is represented as a Cobb-Douglas function as

$C^*= f\ (P_{fert,}\ P_{labr,}\ P_{land,}\ P_{trac,}\ V^*_{it})$ (10)

Where $P_{xi's}$ are the average prices of the inputs and V^*_{it} the value added for the year t adjusted for the stochastic noise captured by Vi where $V_{it}^* = V_{it}\text{-}V_i$.

The multiplicative form of the SFCF is specified as

$$C^*=KP_{fertt}^{a1}\ P_{labrt}^{a2}\ P_{landt}^{a3}\ P_{tract}^{a4}\ Va^{*\theta}it\ e^{vi+ui} \quad (11)$$

The analytically derived cost equation is presented in log form:

$$InC^*=InK+a_1l_nP_{fert\ t}+a_2l_nP_{labr\ t}+a_3l_nP_{land\ t}+a_4l_nP_{trac\ t}+\theta lnVa^*it+Vi+Ui \quad (12)$$

The least expensive cost involved in producing V_{ait} is C*. All of the parameters of the SFCF are obtained from those of the SFPF due to the self-dual nature of the SFPF and SFCF, which allows the duality theory to be supported (Taylor *et al.*, 1986; Jeffrey & Xu, 1998; and Rahji, 2003; 2019).

2.3. Stochastic Frontier Cost Function

Following Taylor *et al.*, (1986) the analytically derived SFCF is represented as a Cobb-Douglas Cost function as

$$C^*= f\ (P_{fert},\ P_{labr},\ P_{land},\ P_{trac},\ V^*_{it}) \quad (13)$$

Where $P_{xi's}$ are the average prices of the inputs and
V^*_{it} the value added for the year t adjusted for the stochastic noise captured by Vi
Where;
$V_{it}^* = V_{it}-V_{i.}$

The multiplicative form of the SFCF is specified as

$$C^*=KP_{fert\ t}^{a1}\ P_{labr\ t}^{a2}\ P_{land\ t}^{a3}\ P_{trac\ t}^{a4}\ Va^{*\theta}it\ e^{vi+ui} \quad (14)$$

The analytically derived cost equation is presented in log form:

$$InC^*=InK+a_1l_nP_{fert\ t}+a_2l_nP_{lab\ t}+a_3l_nP_{lan\ t}+a_4l_nP_{trac\ t}+\theta lnVa^*it+Vi+Ui \quad (15)$$

The least expensive cost involved in producing V_{ait} is C*. All of the parameters of the SFCF are obtained from those of the SFPF due to the self-dual nature of the SFPF and SFCF, which allows the duality theory to be supported (Taylor *et al.*, 1986; Jeffrey & Xu, 1998; and Rahji, 2003; 2019).
The Output Oriented TE is produced by the stochastic frontier production function as
$TE = \frac{Q}{O*} = \frac{Va}{Va*}$.
The analytically derived stochastic frontier cost function is used to obtain the output-oriented economic efficiency as $\frac{C*}{C}$.
Where C is the actual cost of production. and
Economic Efficiency = $\frac{C*}{C}$.; where o<EE<1.
But cost efficiency = $\frac{1}{EE}$; CE $\geq$1
Farrel (1957) established that TE.AE=EE. Hence, AE =EE/TE; 0<AE$\leq$ 1
In this way, the output-oriented TE, AE, and EE are obtained.

2.4 Test for Time Series Regression

Most time series regression analysis requires that unit-root tests be carried out to check for the stationarity of the variables. This was evaluated using the Augmented Dickey-Fuller

(ADF) test. The lag length was also chosen based on the VAR Lag Order Selection criterion. The study used the Trace and Maximum Eigenvalue Rank Test to check for co-integration. Annual time series data on the agricultural sector relating to the period 1960-2021 were used in this study. The ADF test was used to check the variables for stationarity.

The cointegrating Regression Durbin-Watson (CRDW) and Engle-Granger/augmented Engle-Granger (EG/AEG) tests were initially the methods used in testing for cointegration. The test serves as a pre-test to avoid spurious nonsensical regression EG/AEG test asserts that if the residuals from the I(1) variables in regression are found to be I(0), then a linear combination of the variables cancels out the stochastic trends. The regression of such variables would be meaningful and not spurious. The time series variables are said to be cointegrated. i.e. they have a long-term relationship among themselves. The traditional regression methodology is thus applicable to the non-stationary time series. Such a regression is known as a cointegration regression. Its slopes are known as the cointegrating parameters. This is the valuable contribution of the concepts of unit root, cointegration, and others to regression analysis (Gujarati, 2003).

The Johansen's (1988) method has supplanted the other methods. It tests for the relationship among a group of variables, where unconditionally, each variable is to be used as a unit root (Dickey *et. al.,* 1991). It tests for the presence of and the number of cointegrating vectors among the integrated variables (Harris, 1995). Using the trace or maximum eigentest statistics. *Ali et al.,* (2014) stated that once Johansen's test confirms cointegration traditional regression analysis can be applied to the unit root variables in level for long-run analysis and at first difference for short-run analysis.

In regression analysis, variables that are not stationary at the level have unit roots. Variables that are stationary at the level are termed I(0) and can be used in regression analysis, The ADF test, which is a univariate test is used to confirm unit root or otherwise. However, if the residuals from the non-stationary variables are found to be I(0), i.e. stationary at level. Then, the linear combination of the non-stationary variables is said to be I(0). This means that the linear combination of the variables cancels out the stochastic trends in them. As a result, a regression of the unit root time series variables on one another would be meaningful, reasonable, not spurious, and non-nonsensical. In this case, the unit root variables are said to be cointegrated. They thus have a long-term or equilibrium relationship among them and are deemed stationary. Hence, the traditional regression methodology applies to the non-stationary time series data based on the outcome of the test on the residuals.

This outcome is the valuable contribution of the concepts of unit root, cointegration, and other analyses to regression analysis. Gujarati, (2003). Granger (1986) noted that a cointegration test is a pre-test to avoid "spurious regression" situations in research. So a regression of non-stationary but cointegrated variables is known as the cointegrating regression. Its slopes are known as the cointegrating parameters.

The older methods (EG and AEG) used in testing for cointegration have been supplanted by the Johansen cointegration test. This method examines the relationship among a group of variables where each variable has a unit root. (Dickey *et al.*, 1991). To apply the Johansen test, the non-stationary variables must have been confirmed to be integrated in the same order usually I(1). Hence, the Johansen (1988) test is contingent on the variables being I(1). With these conditions met, the test uses the I(0) variables to identify the presence of and the number of cointegrated equations among the integrated variables (Harris, 1995). The confirmation of cointegration by the text implies mean-reversion by the variables. Ali (2014) asserted that once, the Johansen test confirms cointegration, the traditional regression analysis can be applied to the unit root variables. This procedure is used to confirm the integration, cointegration (mean-reversion), and stationarity of the variables used in the current study.

3. Results and Discussion

3.1 Unit Root Test Result

A unit root test for each of the variables in the model is conducted, using Augmented Dickey-Fuller (ADF, 1979) as the most popular test for stationary. The outcome of the unit root test is explained in Table 1. At first difference, the unit root test was disregarded at the 5% significance level. These findings demonstrate that all the series are integrated at order I (1). They supposedly exhibit a stochastic trend.

Table 1. Unit Root Test Results

Variable	Level	1st Difference
LnQ	-2.6184	-4.3988*
Lnfert	-2.5469	-3.9581*
Lnlabr	-1.2845	-3.5366*
Lnland	-2.9413	-5.0065*
Lntract	-1.8352	-4.1507*

Source: Source: Author's computation

Since the series are of order I(I), it is necessary to know the best lag length and carry out the Johansen cointegration test. To investigate, the possible long-run relationships that exist among the series.

3.2 VAR Lag Order Selection Test

The test indicates that the best lag length suggested by AIC criteria is two, and the study incorporates two lags in determining the co-integration between variables.

Table 2. VAR Lag Order Selection

Lag	LogL	LR	FPE	AIC	SC	HQ
0	170.46	NA	1.19e-09	-6.36	-6.18	-6.29
1	530.52	637.03	3.02e-15	-19.25	-18.12*	-18.82
2	565.85	55.72*	2.08e-15*	-19.65*	-17.58	-18.86*
3	576.66	14.97	3.86e-15	-19.10	-16.10	-17.95
4	596.75	23.95	5.37e-15	-18.91	-14.97	-17.40

Source: Author's computation (2022)

Note: * Indicates lag order selected by the criterion. LR: sequential modified LR test statistic (5%). FPE: Final prediction error. AIC: Akaike information criterion. SC: Schwarz information criterion. HQ: Hannan-Quinn information criterion. Endogenous variables: LNGDP, LNfert, LNlabour, LNland, LNtract.. Exogenous variables: C

3.3 Unrestricted Co-integration Rank Test (Trace)

The unrestricted trace test was carried out to show the possibility of co-integrating vectors with trace statistics, providing compelling support for the long-run connection.

Table 3: Unrestricted Co-integration Rank Test (Trace)

No. of CE(S) Hypothesized	Eigenvalue	Trace statistic	0.05 critical value	Prob.**
None *	0.478336	79.72656	69.81889	0.0066
At most 1	0.369924	45.23777	47.85613	0.0863
At most 2	0.182022	20.75632	29.79707	0.3730
At most 3	0.151146	10.10759	15.49471	0.2725
At most 4	0.026484	1.422600	3.841466	0.2330

Source: Author's computation
Note: The Trace test indicates 1 co-integrating equation at the 0.05% level**.** *Denotes rejection of the hypothesis at the 0.05% level**.** ** Mackinnon-Haug-Michelis (1999) p-value

The result from the table shows that there is only one co-integrating equation, as the trace statistic value is greater than the critical value, and the probability value is less than 5 percent.

3.4 Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

The maximum Eigenvalue text reveals the four co-integrating vectors in the model, which provide compelling proof that the model's variables are correlated over the long term. The paper now discusses the model's long-run coefficients.

Table 4. Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

No of CE(s) Hypothesized	Eigenvalue	Max eigen statistic	0.05 critical value	Prob**
None*	0.478336	34.48880	33.87687	0.0422
At most 1	0.369924	24.48145	27.58434	0.1188
At most 2	0.182022	10.64872	21.13162	0.6822
At most 3	0.151146	8.684995	14.26460	0.3133
At most 4	0.026484	1.422600	3.841466	0.2330

Source: Author's computation
Note: The Max-eigenvalue test indicates 1 co-integrating equation at the 0.05 level. Denotes rejection of the hypothesis at the 0.05 level

3.5 The Estimated Stochastic Frontier Production Function.

From the computation of data, the result of the estimated stochastic frontier production function (SFPF) indicates that the lambda (λ) which is the ratio of standard deviations of σu and σv has a value of 1.8853 which is greater than one. It also indicates the appropriateness of the required distributional assumptions for the decomposed error term. The sigma (σ) of 0.3537 is large and significantly different from zero at the 5% level. This also indicates a good fit for the estimated model. The estimated parameters of the SFPF have the anticipated positive sign. They also indicate the expected magnitude of between zero and one, for the estimated function is Cobb-Douglas technology-based. The log-likelihood function indicates a satisfactory fit for model specification. Fertilizer (0.2634), labour (0.3159), land (0.1846), tractor (0. 1587). The result of the positive coefficient of fertilizer agrees with the findings of Usman (2015), Ehimirin et.al, (2016), and Chikezie et.al, (2020). But against the negative coefficient of labour (- 0.2437) and capital (0.0122) Chikezie et. al., (2020). The estimated variance of the one-sided error terms (σu^2) is found to be 0.0976 and that of the statistical noise (σv^2) is 0.0275. The sum of these variances gives sigma² (σ^2). The output-oriented technical efficiency (γ) is 0.7802. The technical inefficiency level is 0.2198. The agricultural sector is thus found to be about 78% output-oriented technically efficient and about 22% output-oriented technically inefficient. The 22% indicates the percentage by which value added (output) in the sector can be increased without increases in the inputs. The estimates of technical efficiency are in line with the findings of other recent studies, like, Mwajombe & Mlozi (2015), Elias et al. (2017), Alam et al. (2012), Ho & Shimada (2019), Asogwa *et al.* (2019). They estimated average efficiency levels of 72% in Tanzania, 78% in Bangladesh, 72% in Ethiopia, 65% in Nigeria, and 72% in India respectively.

The returns to scale is the sum of the coefficients of the estimated stochastic frontier production function, it has a value of 0.9226. The returns to scale of 0.9226 is confirmed as X^2cal is 9.61 while the X^2tal, is 3.841 at the 5% level. This is less than 1 at 0.9226 indicating decreasing returns to scale. This corroborates Wei (2014); Mwajombe & Mlozi (2015), Ho & Shimada (2019), Wondimu & Hassen (2014).

Table 5: The estimated stochastic frontier production function

Variable	Coefficient	Std. Error	Z	P/Z/.
In fert	0.2634	0.0960	2.7438	0.0000
In labr	0.3159	0.0551	5.7332	0.0000
In land	0.1846	0.0896	2.0603	0.0211
InTrac	0.1587	0.0798	1.9887	0.0524
Constant	2.6375	1.2478	2.1137	0.0203

Fertilizer (kg), labour (man-days), land (hectare), tractor (number)
Source: Author's computation

3.6. The Analytically Derived Stochastic Frontier Cost Function.

The analytically derived stochastic cost function is presented as;

$$InC^* = -1.5659+0.2855InP_{fert}+0.3424InP_{labr}+0.2001InP_{land}+0.1720InP_{trac}+1.0839InVa^*$$

In this, average prices of the inputs derived from time series data were used. Using the cost of production and the frontier cost (C*) values, the economic efficiency is obtained as 0.5843 while the economic inefficiency level is 0.4157. The sector is found to be about 58% economically efficient and 42% economically inefficient. The proportion of the minimal cost to the actual cost of production is represented by the estimated mean EE of 58%. This means that reducing the other two production inefficiencies (Technical efficiency and Allocative efficiency) might result in significant cost savings of 42% for the industry. The cost efficiency (CE) defined as the observed/actual cost to the corresponding minimum /frontier cost given the available technology is 1.7115. This is the same as the inverse of economic efficiency (EE). It indicates that, on the age, the sector incurs costs that are about 71% of the cost of production in comparison to the best-practice year.

The scale effect, which is defined as the inverse of the cost elasticity for output, is 0.9226, and equals 1.0839. This is equal to the return to scale in the stochastic frontier production function. The scale effect of less than one means that a 1% increase in cost increases value added by 0.92%. Since the scale effect is less than one there are diseconomies of scale in the sector. This means that costs increase by a greater amount than value added and production is characterized by decreasing returns to scale. The scale effect and the return to scale are thus equivalent. This is so if and only if the output (value added) is homothetic, an assumption that applies to and is implicit in a Cobb-Douglas function structure Chambers (1988) since the return to scale equals scale effect, this assumption is imposed in this study was done by Paudel & Matsuoka (2009). Allocative efficiency refers to the ability of a sector to produce at a given level of value-added using the cost-minimizing input ratio. The economically efficient cost to the technically efficient cost is equal to 0.7489. The sector is thus about 75% allocative efficient. The allocative inefficiency stands at 0.2511 meaning the sector is about 25% allocative inefficient.

The output-oriented technical efficiency is generated by the stochastic frontier production function as Q/Q*. The technical efficiency is given as 0.7802, while technical inefficiency is 0.2198.

The economic efficiency is obtained from the combination of the analytically derived stochastic frontier cost function and the raw cost values for the inputs and the prices of the inputs

EE=C*/C

Va* is generated from stochastic frontier production function Since TE is known.

$Va^* = \frac{Va}{TE}$

Economic Efficiency (EE) = 0.5843. Economic inefficiency (EIE) is 1- EE. Meaning 1- 0.5843 = 0.4157. This indicates the percentage reduction in cost associated with the removal of all inefficiency.

Table 6. Analytically Derived Stochastic Frontier Cost Function

Variable	Coefficient
P fert	0.2855
P labr	0.3424
P land	0.2001
P tract	0.172
Adj. output(θ*)	1.0839
Const.	-1.5659

Note: Mean Technical efficiency = 0.7802. Mean Economic efficiency = 0.5843; C^*_{1t}/C_{1t} ; EE < TE

4. Conclusion and Recommendations

This paper used the stochastic frontier analysis on time series data on the Nigerian agricultural sector to compute the production efficiency measures over the period 1960-2021. A Cobb- Douglas functional form was used which imposed the assumptions of cost elasticity and economies of scale on the model.

The sector is found to be about 78% output-oriented technically efficient and 22% technically inefficient. The 22% indicates the percentage by which value-added can be increased without increases in the inputs. Oni et al., (2009) that generally agricultural sectors are characterized by technical inefficiency, which is 0.1754 in this case, confirm this. The sector is thus about 83% technically efficient and 17% technically inefficient.

The sector is about 75% allocative efficient and 25% allocative inefficient. The sector can improve its resource allocation efficiency by 25% with improvement in AE and elimination of the allocative inefficiency in production.

Economic efficiency stands at 0.5847 while economic inefficiency is 0.4157. The sector is about 58% economically efficient and about 42% economically inefficient. The 58% EE implies that significant cost savings of 42% are possible with improvement in TE and AE. In Farrel's (1957) methodology, it is established that EE is the product of TE and AE. It, therefore, makes sense that economic inefficiency arises from a combination of TE and AE

The cost efficiency index of 1.7115 indicates that the sector incurs costs that are about 71% above the minimum cost defined by the cost frontier as one. In comparison to the best-practice year, the sector wastes about 71% of the cost of production. The scale effect of 0.9226 indicates that a 1% increase in cost increases value-added by 0.92%.

The research purpose of this study is to find means of greatly reducing production inefficiencies in the agricultural sector. This can be achieved by increasing output without necessarily increasing inputs used in the sector. The study therefore recommended that; efficient use of technology be encouraged, and enriched; resource-use efficiency of the existing inputs be improved, and the TE and AE should be enhanced since they have a direct bearing on EE, and subsequently on CE.

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